

Parameter and Payload Identification of a 2-DOF Robotic Manipulator: An Algebraic Identification Approach

Identificación de parámetros y carga útil de un manipulador robótico 2-DOF: un enfoque de identificación algebraica

Julián Y. Páez-García^{ab}, Laura C. Vargas-Tenjo^{ac}, Ricardo A. Charry-Belén^{ad}, Horacio Coral-Enriquez^{ae}

^a Grupo de Investigación SOLSYTEC, Programa de Ingeniería Mecatrónica, Universidad de San Buenaventura (Bogotá), Colombia

^b jypaez@academia.usbbog.edu.co | <https://orcid.org/0000-0003-2119-4994>

^c lcvargas@academia.usbbog.edu.co | <https://orcid.org/0000-0001-7775-7245>

^d rcharry@academia.usbbog.edu.co | <https://orcid.org/0000-0002-1920-1449>

^e hcoral@usbbog.edu.co | <https://orcid.org/0000-0002-1091-9112>

Citation: Páez-García, J., Vargas-Tenjo, L.C., Charry-Belén, R. A., y Coral-Enriquez, H. (2023). Parameter and Payload Identification of a 2-DOF Robotic Manipulator: An Algebraic Identification Approach. *Mutis*, 13(2).1- 14. <https://doi.org/10.21789/22561498.1978>

Recibido: 15 de diciembre de 2022

Aceptado: 1 de febrero de 2023

Copyright: © 2023 por los autores. Licenciado para *Mutis*. Este artículo es un artículo de acceso abierto distribuido bajo los términos y condiciones de la licencia Creative Commons Attribution (<https://creativecommons.org/licenses/by/4.0/>).

ABSTRACT

This paper addresses the problem of identifying the parameters and the payloads of a 2-DOF robotic manipulator. The methodology proposed for our research was the algebraic identification method conducted in two stages: First, identifying the parameters of the manipulator and second, identifying the tip payload. Two different numerical simulation cases were used to validate the proposed identification methodology. In both cases, fast convergence was achieved with a low error percentage.

Keywords: Algebraic identification; Parameter estimation; Payload estimation; Derivative extended-state observer; 2-DOF robotic manipulator; engineering.

RESUMEN

Este artículo aborda el problema de identificar los parámetros y las cargas útiles de un manipulador robótico 2-DOF. La metodología propuesta para nuestra investigación fue el método de identificación algebraica realizada en dos etapas: primero, identificando los parámetros del manipulador, y segundo, identificando la carga útil de la punta. Se utilizaron dos casos de simulación numérica diferentes para validar la metodología de identificación propuesta. En ambos casos se logró una rápida convergencia con un bajo porcentaje de error.

Palabras clave: identificación algebraica; estimación de parámetros; estimación de carga útil; observador de estado extendido derivado; manipulador robótico 2-dof; ingeniería.

INTRODUCTION

Nowadays, robotic manipulators are commonly used in industry due to their ability to reach long distances inside the workspaces where they are operated using various positions and orientation configurations (Liu, 2020). They have been implemented in broad industrial fields such as welding, machining of mechanical parts, assembly lines, load transport, among others, thanks to their unique characteristics

(Chandan et al., 2021; Grau et al., 2017). Robotic manipulators have become popular due to their wide uses, but also because researchers find it challenging develop and implement control strategies for such complex nonlinear systems.

There are many control strategies used for robotic manipulators: simple controllers such as PID (Li & Yu, 2011) and MPD (Huang et al., 2021), advanced systems such as the GPI controller (Becedas et al., 2009), the H_∞ controller (Guo et al., 2015), the Adaptive sliding mode control (ASMC) (Eltayeb et al., 2019) and the Super Twisting Algorithm (Cruz et al., 2018). The last two face the problem of chattering in traditional sliding mode control (SMC). However, all the control strategies mentioned above admit a certain level of uncertainty within the dynamic model of the system. Such uncertainty must be kept at a minimum to avoid the degradation of the control loops (Becedas et al., 2009).

Consequently, in order to reduce the model's uncertainty, researchers have implemented identification strategies, such as the least squares technique (Hashemi & Werner, 2014), orthogonal functions (Ghanbari & Abbasi, 2017), algorithms based on frequency response function (Ferreira & de Oliveira Serra, 2012), the grey-box identification (Coral-Enriquez et al., 2021) and the algebraic identification method (Sira-Ramírez et al., 2014).

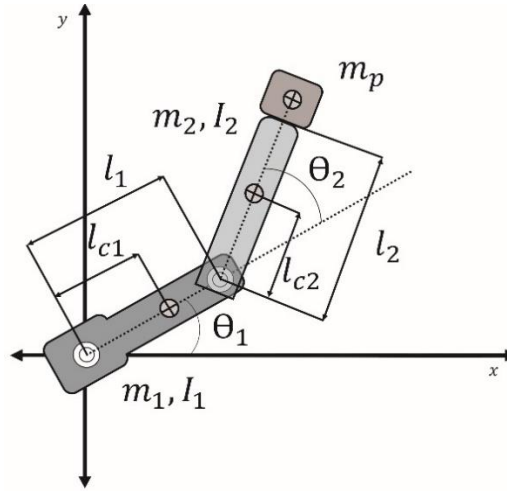
This article focuses on estimating the parameters and the payload of a 2-DOF robotic manipulator using the algebraic identification method. The main contributions of this work are:

- Proposing a step-by-step parameter estimation methodology based on algebraic identification for a 2-DOF robotic manipulator.
- Proposing a step-by-step payload estimation methodology using algebraic identification for a 2-DOF robotic manipulator.
- Developing a state observer capable of estimating the angular velocity of the links that are part of the manipulator without prior knowledge of the manipulator's mathematical model.

The rest of the document is organized as follows: Section II presents the 2-DOF manipulator model. Section III presents illustrates the formulation of the methodology for identification. Section IV is devoted to the results of the simulation. Finally, conclusions and future research are presented in section V.

DYNAMIC MODEL OF A 2-DOF ROBOTIC MANIPULATOR

Figure 1. Robotic system scheme.



Source: own elaboration.

This paper presents the way in which the algebraic identification method was used to estimate the parameters and the payload of a robotic manipulator (see figure 1). The dynamic equation of the manipulator is given by (Lin, 2007):

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (1)$$

in which

$$\begin{aligned} M_{11} &= I_1 + I_2 + m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2 + 2m_2 l_1 l_{c2} \cos(\theta_2) + m_p l_1^2 \\ &\quad + m_p l_2^2 + 2m_p l_1 l_2 \cos(\theta_2), \\ M_{12} &= I_2 + m_2 l_{c2}^2 + m_2 l_1 l_{c2} \cos(\theta_2) + m_p l_2^2 + m_p l_1 l_2 \cos(\theta_2), \\ M_{21} &= I_2 + m_2 l_{c2}^2 + m_2 l_1 l_{c2} \cos(\theta_2) + m_p l_2^2 + m_p l_1 l_2 \cos(\theta_2), \\ M_{22} &= I_2 + m_2 l_{c2}^2 + m_p l_2^2, \\ V_1 &= -m_2 l_1 l_{c2} (2\dot{\theta}_1 + \dot{\theta}_2) \dot{\theta}_2 \sin(\theta_2) - m_p l_1 l_2 (2\dot{\theta}_1 + \dot{\theta}_2) \dot{\theta}_2 \sin(\theta_2), \\ V_2 &= m_2 l_1 \dot{\theta}_1^2 l_{c2} \sin(\theta_2) + m_p l_1 \dot{\theta}_1^2 l_2 \sin(\theta_2), \\ U_1 &= b_1 \dot{\theta}_1, \\ U_2 &= b_2 \dot{\theta}_2, \\ W_1 &= m_1 g l_{c1} \cos(\theta_1) + m_2 g [l_1 \cos(\theta_1) + l_{c2} \cos(\theta_1 + \theta_2)] \\ &\quad + m_p g [l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)], \\ W_2 &= m_2 g l_{c2} \cos(\theta_1 + \theta_2) + m_p g l_2 \cos(\theta_1 + \theta_2), \end{aligned} \quad (2)$$

in which I_1 and I_2 denote the links' moments of inertia, m_1 and m_2 are the masses of link 1 and 2, l_{c1} and l_{c2} refer to the length from the pivot axis of the respective links to their center of gravity, l_1 and l_2 express the lengths of the links, while b_1 and b_2 represent the viscosities, m_p denotes the payload, θ_1 and θ_2 are the positions of the links, and τ_1 and τ_2 express the torques.

FORMULATION OF THE IDENTIFICATION METHODOLOGY

The algebraic identification method was conducted in two stages. First, the parameters of the manipulator were estimated. Then, in accordance with the identified parameters, the payload of the manipulator was identified.

The following assumptions were considered for the algebraic identification: a) the known torque input, b) the position of links θ_1 and θ_2 , which are the only measured output variables, and c) the estimated link velocities for both stages mentioned above.

Parameter identification

Table 1 shows replacements of parameter sets by constants, considering manipulator equations without load ($m_p = 0$). They are useful to facilitate the handling of the equations.

Table 1. Replacement of parameters by constants for the first stage.

Constant		Parameters
k_1	→	$m_2 l_1 l_{c2}$
k_2	→	$m_2 g l_{c2}$
k_3	→	$I_2 + m_2 l_{c2}^2$
k_4	→	$m_1 g l_{c1} + m_2 g l_1$
k_5	→	$I_1 + m_1 l_{c1}^2 + m_2 l_1^2$
k_6	→	b_1
k_7	→	b_2

Source: own elaboration.

Considering equation (1), it is convenient to include an identifier for each equation in order to estimate all the parameters of the manipulator.

By formulating the first equation (1), it is obtained that

$$\tau_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{\theta}_2 + V_1 + U_1 + W_1,$$

$$\begin{aligned} \tau_1 = & [k_5 + k_3 + 2k_1 \cos(\theta_2)]\ddot{\theta}_1 + [k_3 + k_1 \cos(\theta_2)]\ddot{\theta}_2 \\ & + [-k_1(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 \sin(\theta_2)] + k_6\dot{\theta}_1 + k_4 \cos(\theta_1) \\ & + k_2 \cos(\theta_1 + \theta_2), \end{aligned}$$

$$\begin{aligned} \tau_1 = & (k_5 + k_3)\ddot{\theta}_1 + 2k_1 \cos(\theta_2)\ddot{\theta}_1 + k_3\ddot{\theta}_2 + k_1 \cos(\theta_2)\ddot{\theta}_2 - 2k_1\dot{\theta}_1\dot{\theta}_2 \sin(\theta_2) \\ & - k_1\dot{\theta}_2^2 \sin(\theta_2) + k_6\dot{\theta}_1 + k_4 \cos(\theta_1) + k_2 \cos(\theta_1 + \theta_2), \end{aligned}$$

therefore, multiplying by t^2 to eliminate the initial conditions and then integrating twice (see the step-by-step process [here](#)),

$$\begin{aligned}
 \int^{(2)} t^2 \tau_1 = & (k_5 + k_3)t^2\theta_1 - 4(k_5 + k_3) \int t\theta_1 + 2(k_5 + k_3) \int^{(2)} \theta_1 \\
 & + 2k_1 \int t^2 \cos(\theta_2)\dot{\theta}_1 - 4k_1 \int^{(2)} \dot{\theta}_1 t \cos(\theta_2) + k_3 t^2 \theta_2 \\
 & - 4k_3 \int t\theta_2 + 2k_3 \int^{(2)} \theta_2 + k_1 \int t^2 \cos(\theta_2)\dot{\theta}_2 \\
 & - 2k_1 \int^{(2)} \dot{\theta}_2 t \cos(\theta_2) + k_6 \int t^2 \theta_1 - 2k_6 \int^{(2)} t\theta_1 \\
 & + k_4 \int^{(2)} t^2 \cos(\theta_1) + k_2 \int^{(2)} t^2 \cos(\theta_1 + \theta_2).
 \end{aligned} \tag{3}$$

Since there are six terms to be estimated ($k_5 + k_3, k_1, k_3, k_6, k_4, k_2$), the last equation is integrated five times in order to obtain a 6×6 linear system. Therefore, the linear equation is expressed as $P_r(t, \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) x_r = q_r(t, \tau_1)$, in which $P_r(t, \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$, $q_r(t, \tau_1)$ and x_r are given by:

$$P_r = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{bmatrix}, q_r = \begin{bmatrix} q_{11} \\ q_{21} \\ q_{31} \\ q_{41} \\ q_{51} \\ q_{61} \end{bmatrix}, x_r = \begin{bmatrix} k_5 + k_3 \\ k_1 \\ k_3 \\ k_6 \\ k_4 \\ k_2 \end{bmatrix}$$

but, due to the length of each element that belongs to the matrices (p_{11}, \dots, p_{66} and q_{11}, \dots, q_{66}), the elements are defined in detail [here](#).

Finally, the estimation was made by using $x_r = [P_r]^{-1} \cdot q_r$. The estimates take the following values due to the singularity that occurs in the matrices P_r at $t = 0$:

$$x_r = \begin{cases} \text{arbitrary} & \text{for } t \in [t_0, t_0 + \varepsilon) \\ [P_r]^{-1} \cdot q_r & \text{for } t \geq t_0 + \varepsilon \end{cases} \text{ being } \varepsilon = 0.2s.$$

On the other hand, to estimate the last missing constant (k_7), the manipulator equations without load are considered again. Taking the constant k_7 as the only unknown term

$$\tau_2 = M_{21}\dot{\theta}_1 + M_{22}\dot{\theta}_2 + V_2 + U_2 + W_2,$$

$$\tau_2 = [k_3 + k_1 \cos(\theta_2)]\dot{\theta}_1 + k_3\dot{\theta}_2 + k_1\dot{\theta}_1^2 \sin(\theta_2) + k_7\dot{\theta}_2 + k_2 \cos(\theta_1 + \theta_2),$$

the expression is multiplied by t^2 , and then integrated twice,

$$\begin{aligned}
 \int^{(2)} t^2 \tau_2 &= k_3 t^2 \theta_1 - 4k_3 \int t \theta_1 + 2k_3 \int^{(2)} \theta_1 + k_1 \int t^2 \cos(\theta_2) \dot{\theta}_1 \\
 &\quad - 2k_1 \int^{(2)} \dot{\theta}_1 t \cos(\theta_2) + k_1 \int^{(2)} \dot{\theta}_1 t^2 \sin(\theta_2) \dot{\theta}_2 \\
 &\quad + k_3 t^2 \theta_2 - 4k_3 \int t \theta_2 + 2k_3 \int^{(2)} \theta_2 \\
 &\quad + k_1 \int^{(2)} t^2 \dot{\theta}_1^2 \sin(\theta_2) + k_7 \int t^2 \theta_2 - 2k_7 \int^{(2)} t \theta_2 \\
 &\quad + k_2 \int^{(2)} t^2 \cos(\theta_1 + \theta_2).
 \end{aligned} \tag{4}$$

Finally, the constant k_7 was estimated by solving equation 4 (4) for the single unknown term after having elapsed $t_0 + \epsilon$, where $\epsilon = 0.2s$, resulting as follows:

$$\begin{aligned}
 k_7 &= \frac{1}{\int t^2 \theta_2 - 2 \int^{(2)} t \theta_2} \\
 &\quad \cdot \left[\int^{(2)} t^2 \tau_2 - k_2 \int^{(2)} t^2 \cos(\theta_1 + \theta_2) - k_3 t^2 \theta_1 + 4k_3 \int t \theta_1 \right. \\
 &\quad - 2k_3 \int^{(2)} \theta_1 - k_1 \int t^2 \cos(\theta_2) \dot{\theta}_1 + 2k_1 \int^{(2)} \dot{\theta}_1 t \cos(\theta_2) \\
 &\quad - k_1 \int^{(2)} \dot{\theta}_1 t^2 \sin(\theta_2) \dot{\theta}_2 - k_3 t^2 \theta_2 + 4k_3 \int t \theta_2 - 2k_3 \int^{(2)} \theta_2 \\
 &\quad \left. - k_1 \int^{(2)} t^2 \dot{\theta}_1^2 \sin(\theta_2) \right].
 \end{aligned}$$

Payload identification

Equation (2) was also used to estimate the payload mass with the same identification process as in the first identifier. However, it is necessary to consider here that the parameters that are shown in Table 1 have already been identified, so the only unknown parameter is the tip-payload. A table of substitution of parameters by constants is proposed again considering the payload as the only unknown term (see Table 2).

Table 2. Replacement of parameters by constants for the second stage.

Constant		Parameters
z_1	→	$m_p l_1^2$
z_2	→	$m_p l_2^2$
z_3	→	$m_p l_1 l_2$
z_4	→	$m_p g l_1$
z_5	→	$m_p g l_2$

Source: own elaboration.

Equation 2(2) also reveals that either of the two equations that comprise it allowed us to estimate the constants that are shown in Table 2. Thus,

$$\tau_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{\theta}_2 + V_1 + U_1 + W_1,$$

$$\begin{aligned} \tau_1 = & (k_5 + k_3 + z_1 + z_2)\ddot{\theta}_1 + 2(k_1 + z_3)\cos(\theta_2)\ddot{\theta}_1 + (k_3 + z_2)\ddot{\theta}_2 \\ & + (k_1 + z_3)\cos(\theta_2)\ddot{\theta}_2 - 2(k_1 + z_3)\dot{\theta}_1\dot{\theta}_2\sin(\theta_2) \\ & - (k_1 + z_3)\dot{\theta}_2^2\sin(\theta_2) + k_6\dot{\theta}_1 + (k_4 + z_4)\cos(\theta_1) \\ & + (k_2 + z_5)\cos(\theta_1 + \theta_2), \end{aligned}$$

multiplying the last expression by t^2 and then integrating twice,

$$\begin{aligned}
 \int^{(2)} t^2 \tau_1 &= (k_5 + k_3 + z_1 + z_2)t^2\theta_1 - 4(k_5 + k_3 + z_1 + z_2) \int t\theta_1 \\
 &+ 2(k_5 + k_3 + z_1 + z_2) \int^{(2)} \theta_1 \\
 &+ 2(k_1 + z_3) \int t^2 \cos(\theta_2)\dot{\theta}_1 \\
 &- 4(k_1 + z_3) \int^{(2)} \dot{\theta}_1 t \cos(\theta_2) + (k_3 + z_2)t^2\theta_2 \\
 &- 4(k_3 + z_2) \int t\theta_2 + 2(k_3 + z_2) \int^{(2)} \theta_2 \\
 &+ (k_1 + z_3) \int t^2 \cos(\theta_2)\dot{\theta}_2 \\
 &- 2(k_1 + z_3) \int^{(2)} \dot{\theta}_2 t \cos(\theta_2) \\
 &+ (k_4 + z_4) \int^{(2)} t^2 \cos(\theta_1) \\
 &+ (k_2 + z_5) \int^{(2)} t^2 \cos(\theta_1 + \theta_2) + k_6 \int t^2\theta_1 \\
 &- 2k_6 \int^{(2)} t\theta_1.
 \end{aligned} \tag{5}$$

Considering that the tip-payload is the only unknown term, a 5×5 linear system is proposed by performing the integration of (5) four times, and the matrices $P_m(t, \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$, $q_m(t, \tau_1, \theta_1)$ and x_m are given by:

$$P_m = \begin{bmatrix} p_{m11} & p_{m12} & p_{m13} & p_{m14} & p_{m15} \\ p_{m21} & p_{m22} & p_{m23} & p_{m24} & p_{m25} \\ p_{m31} & p_{m32} & p_{m33} & p_{m34} & p_{m35} \\ p_{m41} & p_{m42} & p_{m43} & p_{m44} & p_{m45} \\ p_{m51} & p_{m52} & p_{m53} & p_{m54} & p_{m55} \end{bmatrix}, q_m = \begin{bmatrix} q_{m11} \\ q_{m21} \\ q_{m31} \\ q_{m41} \\ q_{m51} \end{bmatrix}, x_m = \begin{bmatrix} k_5 + k_3 + z_1 + z_2 \\ k_1 + z_3 \\ k_3 + z_2 \\ k_4 + z_4 \\ k_2 + z_5 \end{bmatrix},$$

due to the length of the matrices (p_{11}, \dots, p_{55} and q_{11}, \dots, q_{51}), see [here](#), and the estimates take the following values:

$$x_m = \begin{cases} \text{arbitrary} & \text{for } t \in [t_0, t_0 + \varepsilon) \\ [P_m]^{-1} \cdot q_m & \text{for } t \geq t_0 + \varepsilon \end{cases} \text{ being } \varepsilon = 0.2s.$$

Finally, the payload value was obtained as an independent parameter through some algebraic operations. Therefore, after the constants k_1, \dots, k_7 were identified in the first stage, the mathematical operations continued as follows:

$$z_2 = x_{m[3,1]} - k_3, \quad z_4 = x_{m[4,1]} - k_4,$$

$$z_1 = x_{m[1,1]} - k_5 - k_3 - z_2,$$

thus,

$$\frac{z_1}{z_4} = \frac{m_p l_1^2}{m_p g l_1} \rightarrow \frac{z_1}{z_4} = \frac{l_1}{g},$$

in which g is known and it denotes the gravity, therefore, $l_1 = \frac{z_1 \cdot g}{z_4}$. Knowing l_1 and having the value of the estimate of z_1 , the estimate of the payload was obtained,

$$z_1 = m_p l_1^2 \rightarrow m_p = \frac{z_1}{l_1^2}.$$

Derivative Observer

Applying algebraic identification requires that θ_1 and θ_2 are known. Because they cannot be measured, they must be estimated. Therefore, two equations were proposed to represent an approximated internal model of the signals θ_1 and θ_2 , after the equations were defined as $\ddot{\theta}_1 = 0$ and $\ddot{\theta}_2 = 0$. Considering this approximation, an augmented matrix was proposed, and the state-space representation was outlined:

$$\begin{aligned} \dot{x} &= Ax, \\ y &= Cx, \end{aligned} \tag{6}$$

in which x is the states vector and it is given by $x = [\theta_1, \dot{\theta}_1, \ddot{\theta}_1, \theta_2, \dot{\theta}_2, \ddot{\theta}_2]^T$, and augmented matrices A y C are given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Consider the model (6), a conventional Luenberger observer was proposed to estimate the velocities of the links ($\dot{\theta}_1$ and $\dot{\theta}_2$), as follows:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + L(y - \hat{y}), \quad \hat{y} = C\hat{x}, \\ \hat{x} &= (A - LC)\hat{x} + Ly, \end{aligned}$$

In which L is the gain vector that determines the observer dynamics for state estimation and is defined to place the eigenvalues of the matrix A_{obs} at

$[-80, -390, -391, -392, -393, -394]$; on the other hand, the matrices of the observer were determined by:

$$A_{obs} = [A - LC], \quad B_{obs} = [L],$$

$$C_{obs} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_{obs} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

in which C_{obs} is defined by the states to be estimated ($\widehat{\theta}_1$ and $\widehat{\theta}_2$).

NUMERICAL RESULTS AND DISCUSSION

In order to validate the algebraic identifiers, two different cases of identification are considered, table 3 shows the selected values for each case:

Table 3. 2-DOF robotic manipulator parameters for cases 1 and 2.

Parameter	Case 1	Case 2	Unit
m_1	0.3674	0.4274	<i>Kg</i>
m_2	0.0814	0.1077	<i>Kg</i>
l_{c1}	0.0813	0.1213	<i>m</i>
l_{c2}	0.0616	0.0857	<i>m</i>
l_1	0.1626	0.1925	<i>m</i>
l_2	0.1232	0.1433	<i>m</i>
I_1	0.0009	0.0011	<i>Kgm²</i>
I_2	0.0018	0.0022	<i>Kgm²</i>
b_1	0.1360	0.1570	<i>Nm/(rad/s)</i>
b_2	0.3072	0.3392	<i>Nm/(rad/s)</i>
g	9.81	9.81	<i>m/s²</i>
m_p	0.3515	0.6315	<i>Kg</i>

Source: own elaboration.

Both cases were simulated using the MATLAB-Simulink software and the Runge-Kutta solver to guarantee a good sampling of the signals obtained in the simulations. A sampling time of 0.001s was selected. Figure 2 shows the results of the parameter convergence.

In order to achieve the convergence of the parameters that were estimated in both identification stages, both $\widehat{\theta}_1$ and $\widehat{\theta}_2$ were obtained as a requirement for (3), (4) and (5). Therefore, it was necessary to implement the derivative observer shown in section III.

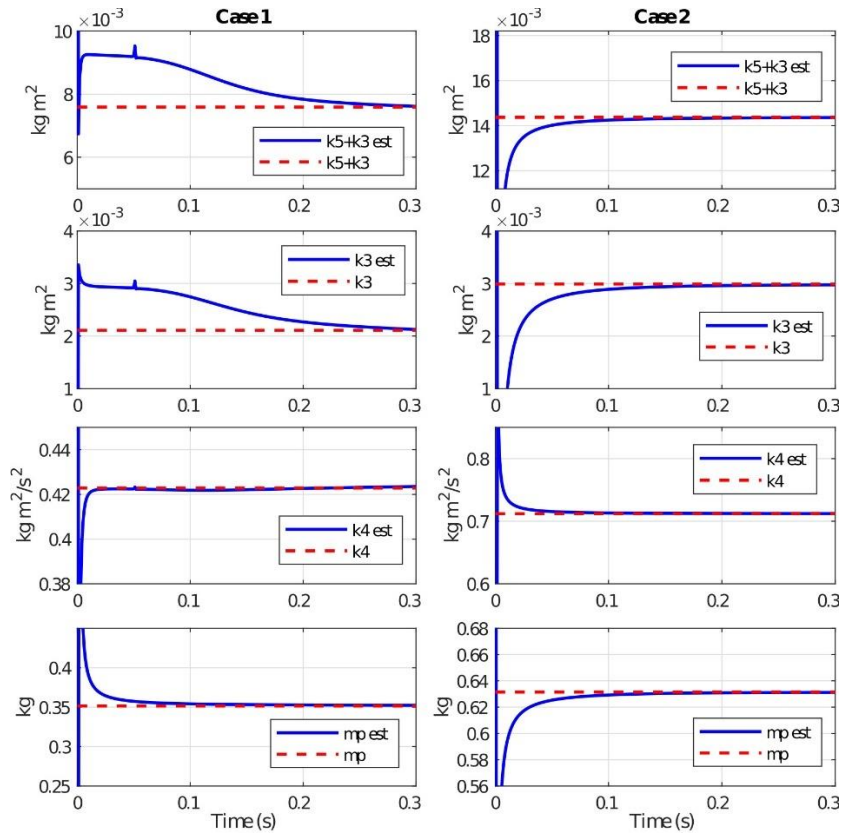
Finally, the convergence was evaluated from the errors obtained between the real values and the estimated values for each of the cases in which the identifiers were tested. The errors are expressed in the following table:

Table 4. Estimation error (%) of 2-DOF robotic manipulator parameters for cases 1 and 2.

Parameter	Case 1 (%)	Case 2 (%)
$k_5 + k_3$	0.02323	0.04004
k_1	0.2449	0.2693
k_3	0.6826	0.5094
k_6	0.008668	0.01314
k_4	0.005106	0.01237
k_2	0.0517	0.06103
$k_5 + k_3 + z_1 + z_2$	0.07454	0.0805
$k_1 + z_3$	0.0405	0.1169
$k_3 + z_2$	0.0455	0.02502
$k_4 + z_4$	0.02889	0.03759
$k_2 + z_5$	0.05145	0.02384
m_p	0.4106	0.0457

Source: own elaboration.

Figure 2. Convergence of some parameters estimated in cases 1 and 2.



Source: own elaboration.

CONCLUSIONS

This paper proposes the two stages of algebraic identification to develop two algebraic identifiers. Those two identifiers were used to estimate the parameters and the payload of a 2-DOF manipulator. The numerical implementation was developed in an open-loop without applying any control scheme, assuming that the torque input was known and considering the position of each link. An observer was proposed to provide estimations of the velocities of each link. Two different simulation cases show that the proposed algorithms converge close to the real parameters, with an error less than 0.6826% in less than 1s.

REFERENCES

- Becedas, J., Trapero, J. R., Feliu, V., & Sira-Ramírez, H. (2009). Adaptive controller for single-link flexible manipulators based on algebraic identification and generalized proportional integral control. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 39(3). <https://doi.org/10.1109/TSMCB.2008.2008905>

Chandan, S., Shah, J., Singh, T. P., Shaw, R. N., & Ghosh, A. (2021). Inverse kinematics analysis of 7-degree of freedom welding and drilling robot using artificial intelligence techniques. In *Artificial Intelligence for Future Generation Robotics*. <https://doi.org/10.1016/B978-0-323-85498-6.00004-6>

Coral-Enriquez, H., Medina-Camacho, S., & Caballero-Mateus, S. (2021). *Modeling and Identification of a 5-DOF Robotic Manipulator Autonomous Bicycles View project Active Disturbance Rejection Control to Stabilize a Bicycle Using a Single-gimbal Control Moment Gyroscope View project*. <https://www.researchgate.net/publication/353206863>

Cruz, G. L., Alazki, H., & Hernández, R. G. (2018). *Super Twisting Control For Thermo's Catalyst-5 Robotic Arm*. 51(13). <https://doi.org/10.1016/j.ifacol.2018.07.295>

Eltayeb, A., Rahmat, M. F., Mohammed Eltoum, M. A., Sanhoury, I. M. H., & Basri, M. A. M. (2019). Adaptive sliding mode control design for the 2-DOF robot arm manipulators. *Proceedings of the International Conference on Computer, Control, Electrical, and Electronics Engineering 2019, ICCCEE 2019*. <https://doi.org/10.1109/ICCCEE46830.2019.9071314>

Ferreira, C. C. T., & de Oliveira Serra, G. L. (2012). An approach for fuzzy frequency response estimation of flexible robot arm from experimental data. *2012 IEEE International Conference on Industrial Technology, ICIT 2012, Proceedings*. <https://doi.org/10.1109/ICIT.2012.6209932>

Ghanbari, M., & Abbasi, M. (2017). *Identification Of Flexible Robot Arm System Using Extended Volterra Series By Kautz Orthogonal Functions*.

Grau, A., Indri, M., lo Bello, L., & Sauter, T. (2017). Industrial robotics in factory automation: From the early stage to the Internet of Things. *Proceedings IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society, 2017-January*. <https://doi.org/10.1109/IECON.2017.8217070>

Guo, Q., Yu, T., & Jiang, D. (2015). Robust H positional control of 2-DOF robotic arm driven by electro-hydraulic servo system. *ISA Transactions*, 59. <https://doi.org/10.1016/j.isatra.2015.09.014>

Hashemi, S. M., & Werner, H. (2014). Parameter identification of a robot arm using separable least squares technique. *2009 European Control Conference, ECC 2009*. <https://doi.org/10.23919/ecc.2009.7074731>

Huang, K., Xian, Y., Zhen, S., & Sun, H. (2021). Robust control design for a planar humanoid robot arm with high strength composite gear and experimental validation. *Mechanical Systems and Signal Processing*, 155. <https://doi.org/10.1016/j.ymsp.2020.107442>

Li, X., & Yu, W. (2011). A systematic tuning method of PID controller for robot manipulators. *IEEE International Conference on Control and Automation, ICCA*. <https://doi.org/10.1109/ICCA.2011.6138081>

Páez-García, J., Vargas-Tenjo, L.C., Charry-Belén, R. A., y Coral-Enriquez, H. (2023). <https://doi.org/10.21789/22561498.1978>

Lin, F. (2007). Robust Control Design: An Optimal Control Approach. In *Robust Control Design: An Optimal Control Approach*. <https://doi.org/10.1002/9780470059579>

Liu, H. (2020). Robot Systems for Rail Transit Applications. In *Robot Systems for Rail Transit Applications*. <https://doi.org/10.1016/B978-0-12-822968-2.01001-9>

Sira-Ramírez, H., García-Rodríguez, C., Cortés-Romero, J., & Luviano-Juárez, A. (2014). Algebraic Identification and Estimation Methods in Feedback Control Systems. In *Algebraic Identification and Estimation Methods in Feedback Control Systems* (Vol. 9781118730607). <https://doi.org/10.1002/9781118730591>